

## EXPERIMENTAL ANALYSIS OF WHISTLE NOISE IN A PARTICLE AGGLOMERATION PIPE

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### ABSTRACT

A self-sustained sound, more usually known as a whistle, refers to a distinct tonal noise created due to the interaction between the sound and flow field. When a positive feedback loop is formed between the two fields, the energy in the mean flow will be transferred into the sound wave, thus giving rise to a whistle. In engineering practice, whistles are destructive as they can produce high sound and vibration levels and may result in risk for mechanical failures. In this work, a flow-related high level tonal noise was found during a measurement on a particle agglomeration pipe, which is a quasi-periodic corrugated structure designed for the exhaust system of heavy-duty trucks. The purpose of the pipe is to enhance particle agglomeration to increase the size of exhaust gas particles. To investigate the origin of the detected tonal noise additional measurements were carried out. Based on the measurement result, the aero-acoustic coupling in the agglomeration pipe was analyzed, revealing that the pipe has a large potentiality to amplify the incident sound power in the presence of a mean flow. Furthermore, the Nyquist stability criterion was applied to confirm the existence of exponentially growing modes in the system at certain conditions.

### MOTIVATION

Based on the research by Katoshevski [1, 2], corrugated pipes, when carefully designed, can modify the inside oscillating flow field so that particles entrained by the flow will experience acceleration or deceleration depending on their relative positions, and therefore lead to the agglomeration or separation of the particles. On the ground of such findings, a new particulate matter (PM) after-

treatment concept, particle agglomeration/grouping, which can shift the size and mass distribution of the PM in vehicle exhaust systems, is proposed. A schematic diagram of a quasi-periodic corrugated pipe designed for the exhaust system of a commercial heavy-duty truck is illustrated in figure 1. From the perspective of acoustics, each unit of the pipe is similar to an expansion chamber, which can reflect the incident sound power when a certain relationship between the wavelength and the chamber length is satisfied. In this sense, the agglomeration pipe may replace part of the muffler package as a compensation for the extra weight and space brought by adding the pipe into the exhaust system. For the sake of investigating the acoustic properties of this agglomeration pipe, a measurement campaign was carried out in the Marcus Wallenberg Laboratory (MWL) at KTH-Royal Institute of Technology.

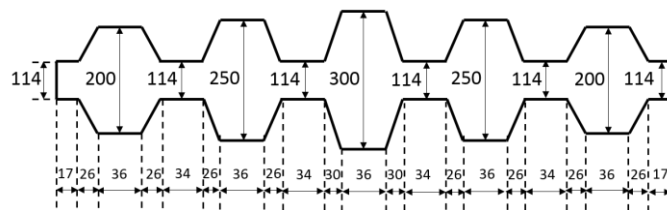


Figure 1. A drawing of the agglomeration pipe used in the measurement campaign.

The measurements focused on determining the two-port of the agglomeration pipe. During the measurement, a clear tonal noise could be heard in the presence of a moderate

mean flow in the test rig, and the spectral property of the tones, presented in figure 2, is related to the flow speed (the two Mach numbers denoted in the figure are used in the two-port test). Given the sharp edges in the pipe where flow separation is prone to happen, the tonal noise might well be flow-induced noise associated with periodic vortex shedding around the edges. Furthermore, if the hydrodynamic mode (regular shedding of vortices) coincides with an available structural/acoustic mode of the pipe, and a positive (unstable) feedback loop is formulated between the two kinds of modes, the flow-induced noise will evolve into a whistle noise, which can lead to unwanted high noise levels.

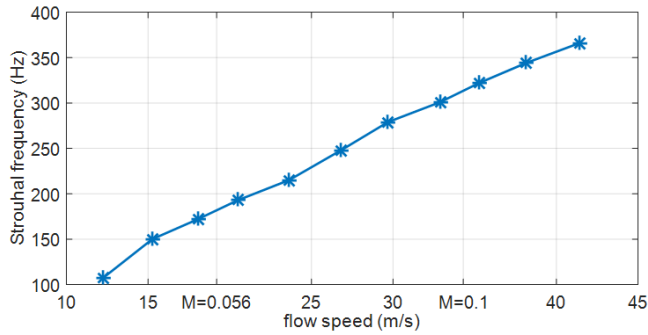


Figure 2. The Strouhal frequency of the flow-induced tones at varying mean flow speeds. The Mach-number used in the two-port test  $M = 0.056$  and  $0.1$  are marked on the x-axis.

To investigate the nature of the tones, a power balance analysis [3,4] was performed based on the two-port measurement data. The analysis revealed that the incident sound power was very likely to be amplified instead of damped in the agglomeration pipe when the interaction between the sound and flow field satisfied certain conditions, i.e., the convection time of the vortex between the up- and downstream edge in a single unit of the pipe matched a multiple of the acoustic period. For further analysis, the reflection at the terminations of the test rig was measured, which enabled the establishment of the eigenfunction of the whole measurement system. The critical zeros of the eigenfunction were determined via the Nyquist stability criterion [4], which confirmed that the flow-induced tones are whistles.

## THEORY

A substantial effort has been put on the investigation of flow-induced noise associated with turbulent shear layers in corrugated structures, with models predicting the spectral distribution as well as other properties regarding whistles proposed [5-7]. However, no available model can be directly used on the agglomeration pipe given its unique (trapezoid cross-section and non-periodic) structure.

Based on previous research on whistling in corrugated

pipes, the generation of the flow-induced noise and whistle noise in the agglomeration pipe can be explained as follows [7].

The time averaged sound power  $\langle P \rangle$  associated with vortex shedding between the up- and downstream edge of a gap (in this case, a single unit of the agglomeration pipe) can be expressed as

$$\langle P \rangle = -\rho_0 \int_V \langle (\boldsymbol{\omega} \times \mathbf{v}) \cdot \mathbf{u}' \rangle dV, \quad (1)$$

where  $\mathbf{v}$  is the mean flow velocity,  $\boldsymbol{\omega} = \nabla \times \mathbf{v}$  is the vorticity,  $\mathbf{u}'$  is the acoustic particle velocity and  $V$  is the volume enclosing the vorticity.

At the upstream edge, a vortex is shed due to an acoustic disturbance and convected downstream with the mean flow. During the first half of the convection path, as illustrated in figure 3,  $(\boldsymbol{\omega} \times \mathbf{v})$  and  $\mathbf{u}'$  are in the same direction, so  $\langle P \rangle$  is negative, i.e., the power of the sound wave is dissipated. On the contrary,  $\langle P \rangle$  becomes positive in the second half, indicating that energy is transferred from the mean flow to the sound wave. As the flow is unstable, the vortex will grow during the convection, thus making the dissipation smaller than the amplification. Therefore, the total effect for the whole convection path is a net sound power increase.

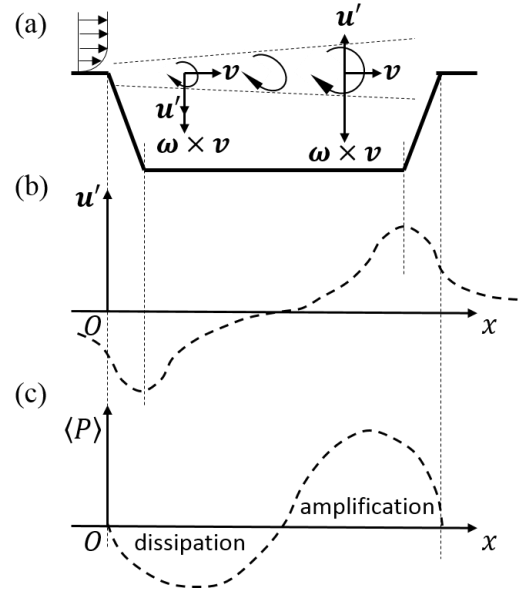


Figure 3. A schematic diagram of (a) the vortex convection between the up- and downstream edge and (b) the instantaneous acoustic velocity distribution as well as (c) the time averaged sound power

The above phenomena leads to a periodic flow separation called a Strouhal-tone which by itself produces sound. Normally this tone is quite weak but it can evolve into whistling. This happens if an available acoustic mode approximately coincides with the Strouhal-tone (“hydrodynamic mode”) and there is a positive feedback loop

between the two modes.

## MEASUREMENT

For the sake of investigating the nature of the tonal noise, a two-port measurement was carried out to experimentally determine the passive property of the agglomeration pipe, based on which the flow-sound interaction in the pipe was analyzed.

### 1. TEST SET-UP

As illustrated in figure 4, the agglomeration pipe was connected to the test rig, whose upstream end was connected to an anechoic chamber where a stable and silent air flow came in, and the downstream end was connected to a muffler to reduce reflections and block external noise. Microphone arrays and loudspeakers were available on both sides of the test section for the determination of the passive property [8]. For the measurement of acoustic signals Brüel & Kjær condenser microphone type 4941, preamplifier type 2670 and conditioning amplifier Nexus type 2690-A, were used. The microphones have been exposed to the same sound field in the calibration tube to eliminate gain and phase difference relative to each other. The synchronous acquisition of the measurement signals and the excitation of the loudspeakers

are controlled by a HP-VXI system. For data acquisition sampling frequency of 12800 was used and 16 averages to suppress random flow noise. Loudspeakers in the up- and downstream were simultaneously driven by using single tone excitation. Moreover, the generated excitation signal was calibrated so that amplitude of the incident acoustic wave was kept constant 1 Pa. The generated excitation signal is amplified by NAD C370 amplifier.

A pitot tube was mounted on the upstream rig to measure the incident flow profile with Swema 3000. The tests were performed with air at 20°C and standard pressure. The temperature of the measurement section has been continuously monitored by attaching the thermo-couple and Digitron 2751-K, the barometric pressure is measured using a Brüel & Kjær barometer type UZ 0001 and the relative humidity is measured with a Rotronic Hygrolog HL-20.

All the geometry of the test set-up can be found in the illustration (the unit is mm). The highest measurable frequency is determined with the diameter of the duct where the frequency of the first nonplanar propagation is 1600 Hz. In order to extend the lower frequency range an extra microphone was used on the inlet and outlet side.

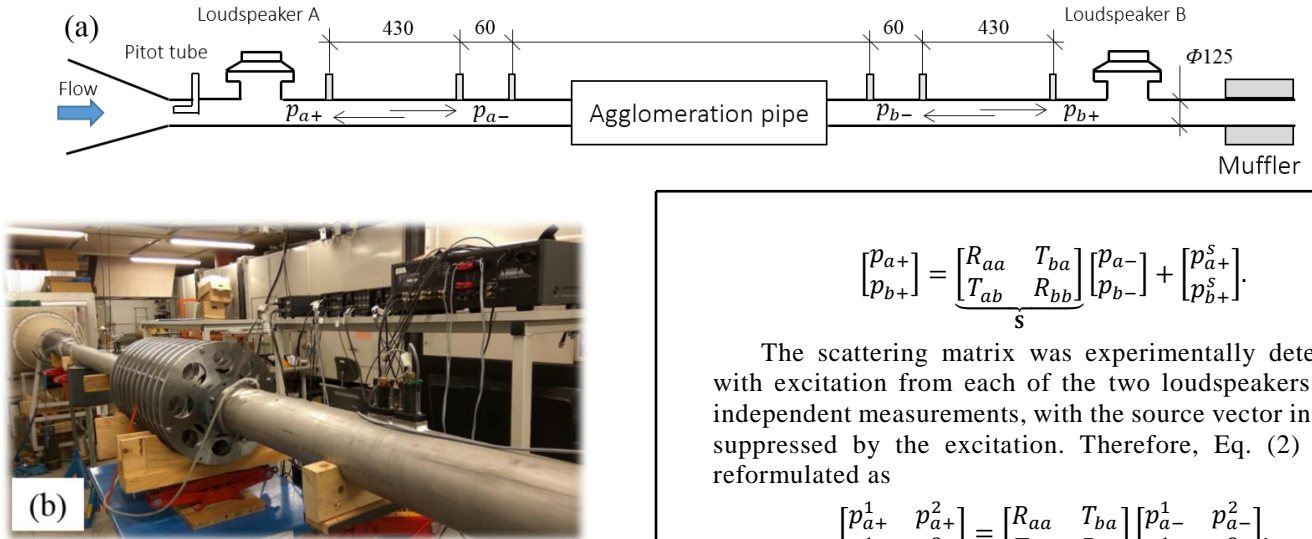


Figure 4. (a) A schematic diagram and (b) photo of the test set-up

### 2. POST-PROCESSING METHOD

The agglomeration pipe, as an acoustic two port, can be acoustically characterized by the scattering matrix  $\mathbf{S}$  [8], which formulates the relationship between the ingoing ( $p_{a-}, p_{b-}$ ) and outgoing ( $p_{a+}, p_{b+}$ ) complex-valued pressure wave amplitudes as well as the potential internal sources ( $p_{a+}^s, p_{b+}^s$ ) in the form

$$\begin{bmatrix} p_{a+} \\ p_{b+} \end{bmatrix} = \underbrace{\begin{bmatrix} R_{aa} & T_{ba} \\ T_{ab} & R_{bb} \end{bmatrix}}_{\mathbf{S}} \begin{bmatrix} p_{a-} \\ p_{b-} \end{bmatrix} + \begin{bmatrix} p_{a+}^s \\ p_{b+}^s \end{bmatrix}. \quad (2)$$

The scattering matrix was experimentally determined with excitation from each of the two loudspeakers in two independent measurements, with the source vector in Eq. (2) suppressed by the excitation. Therefore, Eq. (2) can be reformulated as

$$\begin{bmatrix} p_{a+}^1 & p_{a+}^2 \\ p_{b+}^1 & p_{b+}^2 \end{bmatrix} = \underbrace{\begin{bmatrix} R_{aa} & T_{ba} \\ T_{ab} & R_{bb} \end{bmatrix}}_{\mathbf{S}} \begin{bmatrix} p_{a-}^1 & p_{a-}^2 \\ p_{b-}^1 & p_{b-}^2 \end{bmatrix}. \quad (3)$$

where the superscript '1' represents the upstream loudspeaker on and '2' the downstream one on.

With the scattering matrix obtained, the sound attenuation ability ("down-stream direction") of the agglomeration pipe can be assessed via

$$TL_{ab} = 10 \log_{10}(1/|T_{ab}|^2). \quad (4)$$

However, what is also interesting in this case is the interaction between the sound and flow field, which can be investigated from the perspective of net energy flux, i.e., amplification or dissipation of the incident sound power in

the agglomeration pipe. Here, two similar but different power balance formulations are used. For the first formulation [3], a new acoustic state variable

$$\mathbf{P}_{p\pm} = \begin{bmatrix} (1 \mp M_a) \sqrt{\frac{S_a}{\rho_{0a} c_{0a}}} \cdot p_{a\pm} \\ (1 \pm M_b) \sqrt{\frac{S_b}{\rho_{0b} c_{0b}}} \cdot p_{b\pm} \end{bmatrix}, \quad (5)$$

which is related to the time averaged sound power

$$\langle P_{p\pm} \rangle = \mathbf{P}_{p\pm}^* \mathbf{P}_{p\pm}, \quad (6)$$

is introduced. Here  $*$  denotes transpose and complex conjugation. On such basis, the net sound power output from the two-port system (agglomeration pipe) can be expressed as

$$\langle P_{out} \rangle = \mathbf{P}_{p+}^* \mathbf{P}_{p+} - \mathbf{P}_{p-}^* \mathbf{P}_{p-} = \mathbf{P}_{p-}^* (\mathbf{S}_p^* \mathbf{S}_p) \mathbf{P}_{p-} - \mathbf{P}_{p-}^* \mathbf{P}_{p-}, \quad (7)$$

where

$$\mathbf{S}_p = \begin{bmatrix} \frac{1-M_a}{1+M_a} R_{aa} & \frac{1-M_a}{1-M_b} \sqrt{\frac{\rho_{0b} c_{0b} S_a}{\rho_{0a} c_{0a} S_b}} \cdot T_{ba} \\ \frac{1+M_b}{1+M_a} \sqrt{\frac{\rho_{0a} c_{0a} S_b}{\rho_{0b} c_{0b} S_a}} \cdot T_{ab} & \frac{1+M_b}{1-M_b} R_{bb} \end{bmatrix}. \quad (8)$$

This implies that the incident power is amplified when  $\langle \overline{P_{out}} \rangle > 0$  and dissipated when  $\langle \overline{P_{out}} \rangle < 0$ . When properly diagonalized and normalized, the net power output (assuming 1W incident power) can be reformulated as

$$\langle \overline{P_{out,1W}} \rangle = \sum_q \lambda_q |p'_q|^2 - 1, \quad (9)$$

where  $\lambda_q$  is an eigenvalue of the Hermitian matrix  $\mathbf{S}_p^* \mathbf{S}_p$ .

In this way, the maximum and minimum normalized potential net power output can be expressed as

$$\langle \overline{P_{out,1}^{max}} \rangle = \lambda_{max} - 1 \text{ and } \langle \overline{P_{out,1}^{min}} \rangle = \lambda_{min} - 1. \quad (10)$$

This first formulation was suggested by Auregan and Starobinsky [3] and finds the incident wave combinations that maximizes or minimizes the sound power from a two-port.

For the second formulation, the normalized power output from the two-port with incident wave from the inlet (superscript ‘a’) and outlet (superscript ‘b’), respectively, is expressed as

$$\frac{P_{out}}{P_{in}^a} = \frac{|R_{aa}|^2 (1-M_a)^2}{(1+M_a)^2} + \frac{|T_{ab}|^2 (1+M_b)^2}{(1+M_a)^2}$$

and

$$\frac{P_{out}}{P_{in}^b} = \frac{|R_{bb}|^2 (1+M_b)^2}{(1-M_b)^2} + \frac{|T_{ba}|^2 (1-M_a)^2}{(1-M_b)^2}. \quad (11)$$

Then the total normalized (1W incident at both a and b) net power output is

$$\langle \overline{P_{out,2}} \rangle = \frac{P_{out}}{P_{in}^a} + \frac{P_{out}}{P_{in}^b} - 2. \quad (12)$$

In comparison, formulation 1 finds the max/min power amplification for correlated inputs, while formulation 2 provides the max power amplification assuming equal uncorrelated inputs. Therefore, the result given by Eq. (12) may fall out of the range given by Eq. (10).

With the flow-sound interaction quantified by the two-port formalism, it is possible to further check if a positive feedback loop between the hydrodynamic and acoustic mode is formed [4]. Here the so-called reflection matrix  $\mathbf{R}$  [4], which is in the form

$$\mathbf{R} = \begin{bmatrix} R_a & 0 \\ 0 & R_b \end{bmatrix}, \quad (13)$$

is introduced. In Eq. (13),  $R_a$  and  $R_b$  are the reflection coefficient of the up- and downstream terminations of the whole measurement system, respectively,

$$R_a = \frac{p_{a-}^2}{p_{a+}^2} \text{ and } R_b = \frac{p_{b-}^2}{p_{b+}^2}. \quad (14)$$

For example,  $R_a$  is the ratio between the ingoing and outgoing wave on the upstream side of the test section with incident sound waves from the downstream loudspeaker only.

One thing worth mentioning here is that compared to the standard procedure for two-port measurements, see e.g. Åbom [8], for this case the reflection coefficients were determined separately, i.e., with the agglomeration pipe removed from the test rig. A direct measurement including the agglomeration pipe showed magnitudes of the reflection coefficients much larger than 1 for certain frequencies, indicating that the data was affected by “non-linear” effects. The measurement of the two-port itself should be correct as long as it was only amplifying sound and the “non-linear” whistling loop was closed by reflections from the boundaries [4].

With the reflection matrix obtained, Eq. (2) can be reformulated as

$$\begin{bmatrix} p_{a+} \\ p_{b+} \end{bmatrix} = \mathbf{SR} \begin{bmatrix} p_{a+} \\ p_{b+} \end{bmatrix} + \begin{bmatrix} p_{a+}^s \\ p_{b+}^s \end{bmatrix}. \quad (15)$$

The source term is independent of the passive property of the two-port system. Thus, the eigenvalues of the system can be obtained from [4]

$$(\mathbf{I} - \mathbf{SR}) \begin{bmatrix} p_{a+} \\ p_{b+} \end{bmatrix} = 0, \quad (16)$$

in which  $\mathbf{I}$  is the unit matrix. Eq. (16) has non-trivial solutions if the determinant  $D = \det(\mathbf{I} - \mathbf{SR})$  is zero, but the critical eigenfrequencies  $\omega$  (zeros in the lower complex plane) are not straightforward to get. Instead, the Nyquist stability criterion is adopted here, which, when applied to positive real frequencies (as is the case here), provides a good estimation of the eigenfrequencies if the negative real axis on the complex plane is crossed by  $D$  or equivalently speaking, the origin of the complex plane is encircled. And such encirclement indicates that the positive feedback loop between the hydrodynamic and acoustic mode is formulated, i.e., a whistle noise is generated.

## RESULT

The incident flow profiles, with the mean flow Mach number at 0.056 and 0.1, respectively, are illustrated in figure 5. Obviously, the flow is turbulent in both cases and certainly the flow speed is in a range where separation will

occur in the corrugated pipe as indicated by the observed Strouhal-tones, see figure 2.

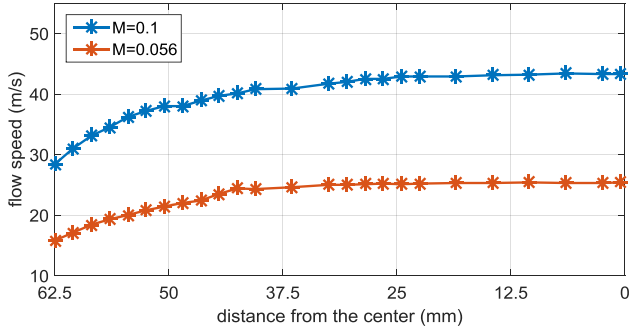


Figure 5. The incident flow profile.

The measured transmission loss (TL) of the agglomeration pipe for one no-flow case and two flow cases are presented in figure 6. As a comparison, the simulation result is also depicted. The simulation was conducted using the linearized potential flow solver in the commercial software COMSOL Multiphysics® [9], with the convective wave equation

$$-ik(k\Phi + M \cdot \nabla\Phi) + \nabla[\nabla\Phi - (ik\Phi + M\nabla\Phi)M] = 0 \quad (16)$$

solved, where  $\Phi$  is the velocity potential. In this solver, the background mean-flow field is assumed to be inviscid, irrotational and incompressible, with the mean-flow speed varying with the pipe cross-section area. The convection effect is imposed onto the sound field, but the details of the flow field cannot be captured. In particular the model will not include vortex-sound interaction effects as described by Eq. (1).

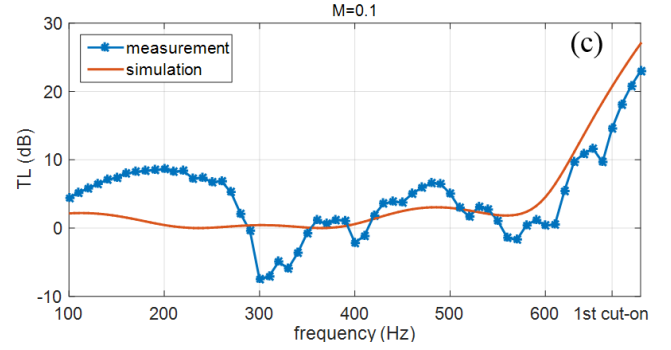
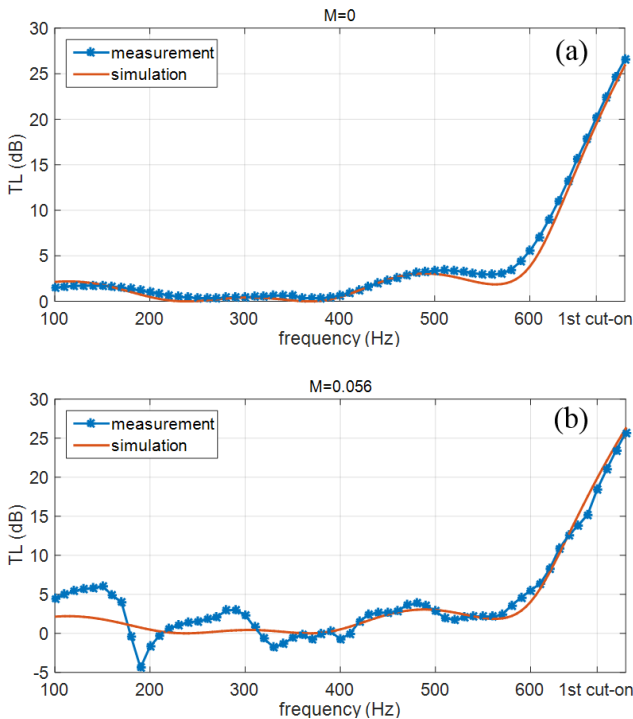


Figure 6. The transmission loss of the agglomeration pipe at different flow speeds.

As shown in figure 6, the TL starts to rise at around 600 Hz and goes beyond 20 dB when above the first cut-on frequency in the corrugated pipe chamber (around 670 Hz). There is as expected a good agreement between measurement and simulation for the no-flow case, but deviations can be found in the two flow cases. Especially at, for instance, approximately 200 Hz in figure 6 (b) and 300 Hz in (c), the measured TL goes negative, which happens to coincide with the audible Strouhal tones for Mach number 0.056 and 0.1, respectively (see figure 2). This is a strong indication that the neglected vortex-sound effects in the numerical model used are important.

The sound pressure level (SPL) in these two flow cases collected by one of the three downstream microphones is presented in figure 7. Nevertheless, the Strouhal tones at around 200 Hz and 300 Hz in the two cases are clearly at a much higher level than the excitation from the loudspeaker (500 Hz for the Mach number 0.056 case and 390 Hz for the Mach number 0.1 case), indicating a strong flow-sound interaction.

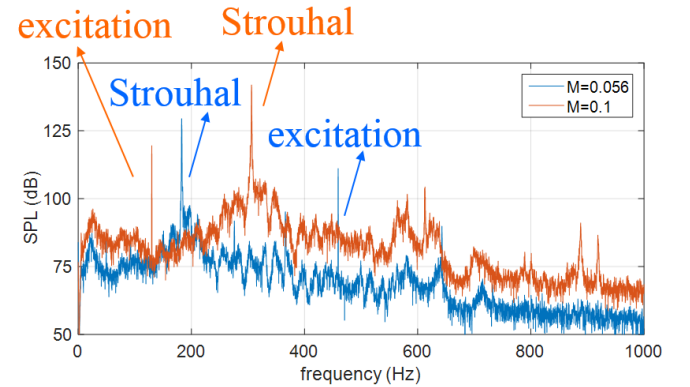


Figure 7. The sound pressure level spectrum on the downstream side of the agglomeration pipe.

Given the information in figure 7, a possible explanation of the negative TL is the net increase of sound power due to flow-sound interaction, i.e., the agglomeration pipe

amplified instead of damped the incident sound wave at certain frequencies. For the sake of checking the validity of such speculation, the power balance of the agglomeration pipe was formulated and illustrated in figure 8.

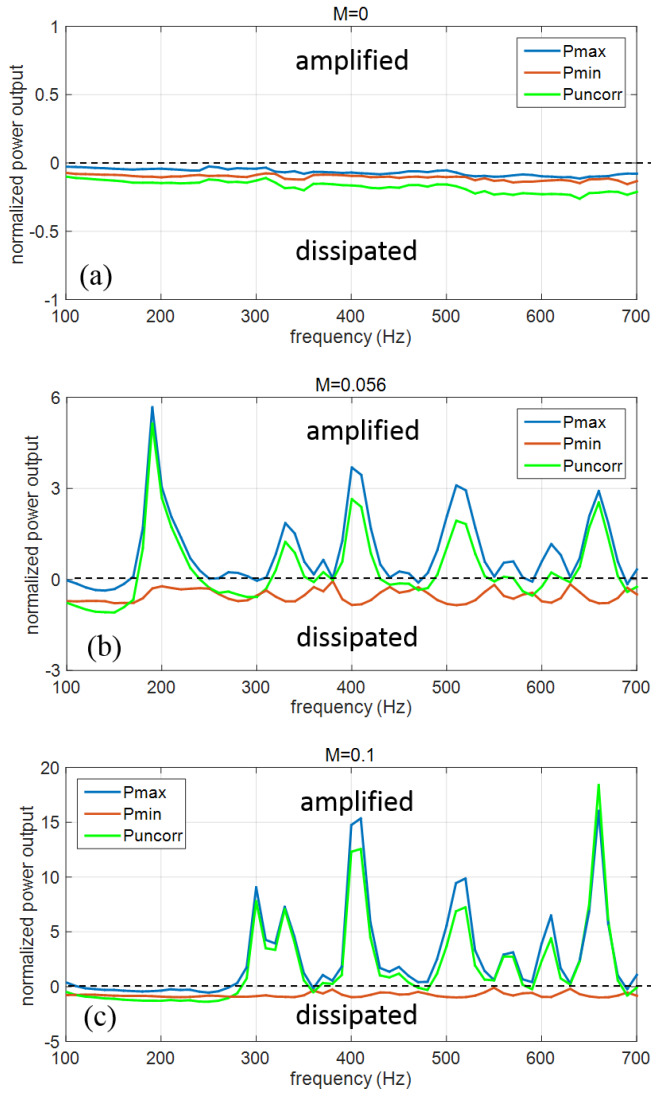
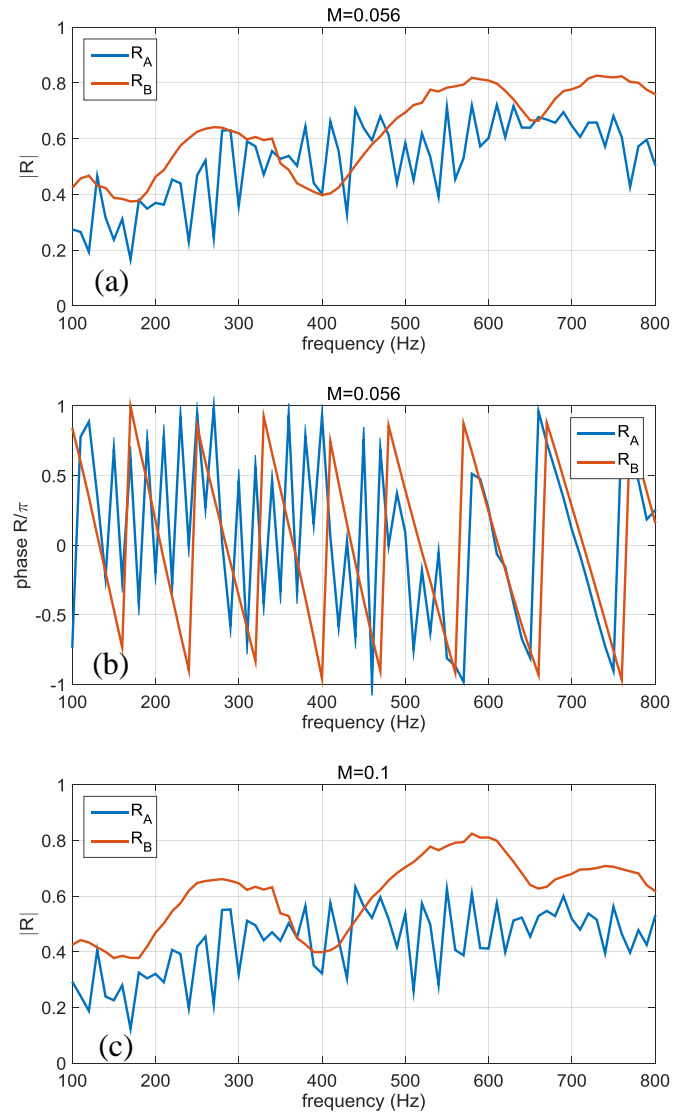


Figure 8. The normalized net power output from the agglomeration pipe. ' $P_{max}$ ' and ' $P_{min}$ ' denote the maximum and minimum potential power output calculated via Eq. (10), and ' $P_{uncorr}$ ' denotes the uncorrelated power output calculated via Eq. (12).

Slight differences can be found between the two formulations, probably caused by the fact that incident waves at the two ports are taken as correlated in formulation 1 but uncorrelated in formulation 2. But the new formulation 2 seems to follow closely the ' $P_{max}$ ' curve for formulation 1. In both the two flow cases, there is a large potentiality for the incident sound power to be amplified, not only for the audible tones (~200 Hz and ~300 Hz, respectively) but also at many other frequencies, indicating that the agglomeration

pipe can serve as a muffler (beyond 600 Hz) and an amplifier at the same time.

An amplified sound power, however, does not necessarily mean the pipe was whistling, and the feedback loop between the hydrodynamic and acoustic mode needs to be checked to draw the conclusion. Here, the magnitude and normalized phase of the reflection coefficient at the inlet and outlet (denoted as 'A' and 'B', respectively) of the test rig are presented in figure 9. Although the two terminations were designed to reduce reflections, there were quite strong reflections that partly also could be created by the loudspeakers.



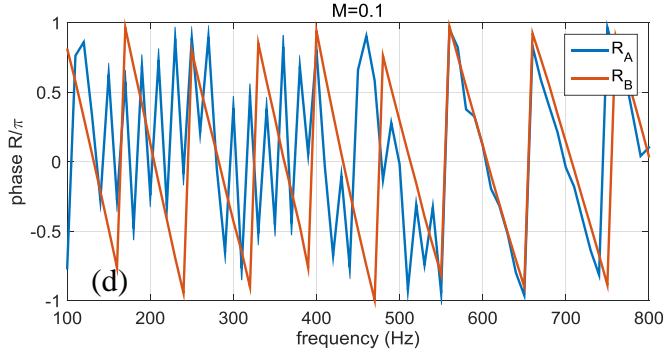


Figure 9. The magnitude and normalized phase of the reflection coefficient at the inlet (A) and outlet (B) of the test rig at different flow speeds.

The Nyquist plot of the determinant for the two flow cases, i.e.,  $D(\omega)$  in a complex plane (see Eq. (16)) are shown in figure 10. In both cases the origin is encircled by the contour, indicating that the system is unstable, i.e., the agglomeration pipe does whistle, and the critical zero emerges between 200 and 220 Hz in the Mach number 0.056 case and 300 and 320 Hz in the Mach number 0.1 case, respectively, which agrees well with the corresponding negative TL in figure 6.

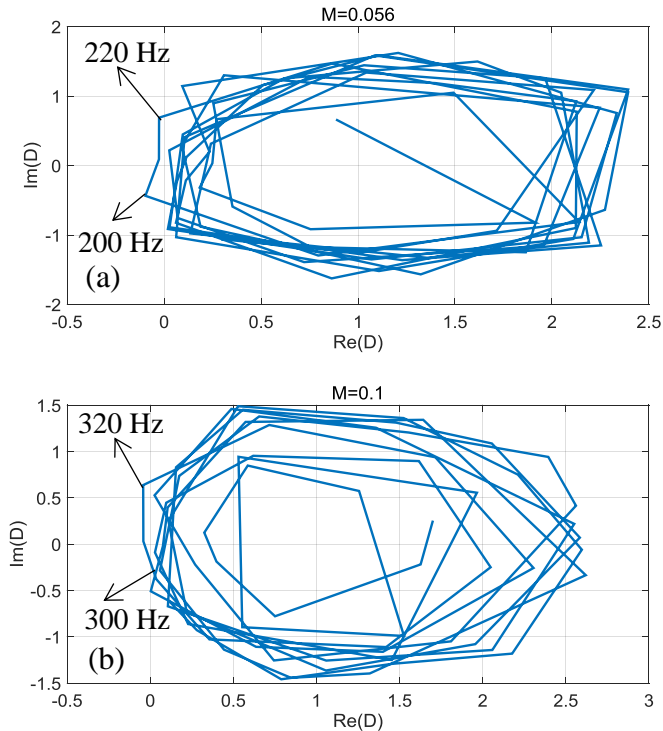


Figure 10. The determinant contour with the application of the Nyquist stability criterion, i.e., the frequencies that enclose the origin.

## CONCLUSION

The flow-induced noise in a quasi-periodic corrugated pipe was experimentally analyzed, with the vortex-sound interaction quantified by the measured two-port data. Two formulations to describe the net power output were provided, both of which showed that the pipe had a large potentiality to amplify the incident sound power. The stability of the system was checked via the Nyquist stability criterion, confirming the noise as a whistle.

The current work is measurement-based, but the formalism used to analyze the flow-sound interaction is available to predict whistling if the passive acoustic properties (the two-port data and termination reflection) can be obtained from simulations, which requires solvers that can solve, for example, the linearized Navier-Stokes equation [10] to capture the details of the flow field, otherwise the flow-generated noise cannot be predicted (like in the current work).

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